Unwillingness to invest during balance-of-payments crises

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Abstract

This paper comes back to the mechanisms at work in recent emerging market crises characterized by large real depreciations and sharp drops in aggregate investment. While most of the existing literature explains the drop in investment by tightened borrowing constraints that make entrepreneurs *unable* to invest, I build a model where entrepreneurs are able but *unwilling* to invest. This model is consistent with the recent empirical evidence which suggests that the observed decrease in lending is mainly due to a lower demand for loans, not supply.

The model studies a two-period small open economy where multiple equilibria associated with different real exchange rates and levels of investment can arise. Two variants of the model are considered, corresponding to the two alternative mechanisms—inability versus unwillingness to invest. Conditions for the existence of multiple equilibria are derived in each case.

Keywords: balance-of-payments crises, liquidity constraints, multiple equilibria.

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1 Introduction

In the last decades emerging economies have experienced several economic collapses centered around balance-of-payments crises. Of particular importance is the behavior of investment during those events. Empirical studies have shown that the real depreciation which characterizes these crises (Kaminsky & Reinhart 1999, Tornell & Westermann 2002, Calvo, Izquierdo & Talvi 2006, Calvo, Izquierdo & Mejía 2004) comes along with a sharp drop in investment (Tornell & Westermann 2002, Calvo et al. 2006). There is no doubt that this drop in investment contributed to amplify the extent of the collapse.

But why don't entrepreneurs invest during crises? A depreciated real exchange rate reduces the cost of domestic inputs and is likely to be followed by a real appreciation in the future. If financial markets were perfect, low investment costs and high expected future proceeds should induce entrepreneurs to invest.

The existing theoretical literature on emerging market crises explains the drop in investment by the *inability* of entrepreneurs to invest during a crisis. In several recent models where imperfections in financial markets lead to credit rationing, a real depreciation tightens the borrowing constraints through a balance sheet effect and provokes a credit crunch (Krugman 1999, Schneider & Tornell 2004, Aghion, Bacchetta & Banerjee 2004*a*). In these models, the decrease in investment entirely comes from a lower supply of loanable funds.[1]

This explanation certainly accounts for important aspects of crises in emerging economies. However, it has some limitations.

First, it is at odd with some recent empirical evidence. Hale & Arteta (2006) identify changes in the supply and demand of foreign credit to private firms in 29 emerging markets after large real depreciations between 1981 and 2004. They show that the large decrease in credit that takes place during the first five months after the depreciation is mainly due to a collapse in

[1]In the papers cited above the credit crunch is the result of a fragility in the firms' balance sheets. In other models the fragility originates in the banking sector. See Jeanne & Zettelmeyer (2002) for a common framework encompassing both approaches.

the demand for credit (by about 30%), while the decrease in supply is much lower (around 8%) but more persistent. This result is consistent with the behavior of investment during the post-crisis recovery phase observed by Calvo et al. (2006) in 22 episodes of emerging market crises between 1980 and 2004. According to these authors, investment grows by about 25% in the first two years of recovery while both domestic credit and external savings do not recover at all. The fact that investment recovers even though financing conditions are still the same as at the heart of the crisis suggests that the lack of finance is not the factor that limits investment during these events. Both works support the idea that at least some fraction of entrepreneurs are able but *unwilling* to invest during balance-of-payments crises.

The aim of the present paper is to provide a simple explanation for the unwillingness of entrepreneurs to invest during a crisis, based on the higher probability of failure of investment projects. It builds a model with borrowing constraints where entrepreneurs producing non-tradable goods start investment projects which can be interrupted by random liquidity shocks. I shall argue that the ability to resist such a liquidity shock depends on the real exchange rate so that the risk of becoming illiquid increases during episodes of real depreciation. With a lower probability of success, projects are less profitable during a crisis and fewer entrepreneurs are willing to start them. These variations in the probability of success constitute a powerful amplification mechanism. As a lower level of investment further depreciates the real exchange rate,[2] the mechanism is self-reinforcing and can even lead to the existence of multiple equilibria. Thus, a very small exogenous shock—indeed, even a purely expectational shock—can trigger a shift from one equilibrium to another and provoke a large crisis.

The second limitation of the credit crunch hypothesis is theoretical. Existing models based on the inability to invest during a crisis rely on an underlying financial accelerator mechanism and only produce multiple equilibria when the borrowing constraints are weak enough.[3] Therefore, they

[2]This inverse causality—from investment to the real exchange rate—is standard in real models with two sectors.

[3]More precisely, a real depreciation affects the level of investment through its adverse

are not valid for economies with a low level of financial development. On the contrary, I will show that multiple equilibria with unconstrained investment arise when the borrowing constraints are strong enough. Thus, contrary to the mechanism modeled by the existing literature, a model of crisis based on the unwillingness to invest is relevant for financially undeveloped economies. To make this point clear, the paper uses a unified framework to study both crisis mechanisms, inability versus unwillingness to invest, and compare their formal properties. In particular, conditions for the existence of multiple equilibria will be derived for the two variants of the model, along with the characteristics of equilibria corresponding to crisis times.

The paper is related to the literature on financial crises in emerging economies that was cited above. In particular, the model can produce multiple equilibria associated with different real exchange rates and different levels of investment, as in Krugman (1999), Schneider & Tornell (2004), and Aghion et al. (2004*a*).

It is also related to the literature on liquidity risks in the corporate sector, initiated by Holmstrom & Tirole (1998). Caballero & Krishnamurthy (2001) adapted the standard liquidity model to the case of an open economy. More recently, Aghion, Angeletos, Banerjee & Manova (2005) explored the macroeconomic consequences of liquidity risk on the investment decisions of entrepreneurs and showed how it affects volatility and growth.

The remaining of the paper is organized as follows. The model is presented in section 2. Section 3 studies the different equilibria of the two model's variants: section 3.3 studies the game where entrepreneurs are unable to invest during crises while section 3.4 studies the alternative game where investment can be unconstrained in crisis equilibria. Section 4 fully solves the latter game in the case of a binomial distribution for the liquidity shocks. Section 5 concludes.

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effect on the cash flow of firms producing non-tradable goods. A given real depreciation leads to a large decrease in investment if investment is very sensitive to changes in cash flow, *i.e.* if the financial multiplier $\partial I/\partial W$ (where I denotes investment and W denotes the cash flow) is large enough. In these models, a large financial multiplier corresponds to a weak borrowing constraint.

2 The model

2.1 Structure of the model

The model is designed to be as close as possible to the existing models cited above, so that its results can be easily compared with theirs. More specifically, this paper abstracts from monetary issues and describes a real economy similar to Krugman (1999) and Schneider & Tornell (2004).

Consider a small open economy with two periods $t = 1$ and $t = 2$. There are two goods: a tradable good T, used as *numeraire*, and a non-tradable good N with price p_t. The price p_t is a measure of the real exchange rate.[4]

There are three types of agents: domestic consumers, domestic entrepreneurs producing non-tradable goods, and foreign lenders. Each type of agent lives on a continuum of measure 1. They have access to an international financial market where they can trade one-period bonds denominated either in tradable or non-tradable goods.

The model consists in a game played by the entrepreneurs. Two alternative setups will be considered for this game. A first variant reproduces the structure of existing models where a real exchange rate crisis is due to the inability of entrepreneurs to invest. This game setup will be referred to as the Constrained Investment game (henceforth CI game) and is presented here for the sake of comparison. The second variant embeds the alternative mechanism discussed in the introduction. It is referred to as the Fear of Illiquidity game (henceforth FI game).

Foreign lenders

Foreign lenders are risk-neutral and have a large enough endowment of T goods in each period to cover any demand for funds from the domestic agents. They value the consumption of tradable goods according to the utility function $U = c_1^T + \mathrm{E}_1[c_2^T]$, where c_t^T is their consumption of T goods in period t and $\mathrm{E}_1[\cdot]$ denotes the expectation conditional on the information available at the beginning of period 1. This sets to unity the riskless rate of

[4] A high value of p_t corresponds to an appreciated real exchange rate.

return of bonds denominated in T goods.[5]

Consumers

Domestic consumers are endowed with e^T (e^N) units of tradable (non-tradable) goods in each period (these endowments are strictly positive) and enter period 1 with an initial wealth ω_1. They maximize the utility function $U = \sum_{t=1}^{2} \mathrm{E}_1[c_t^T + d \log c_t^N]$, with obvious notations, subject to the intertemporal budget constraint $\sum_{t=1}^{2} \mathrm{E}_1[c_t^T + p_t c_t^N] \leq \omega_1 + 2e^T + \big(p_1 + \mathrm{E}_1[p_2]\big)e^N$. It is assumed that their endowments and initial wealth are large enough so that this maximization program has an interior solution. Their net demand for non-tradable goods at time t is then given by

$$D_t = \frac{d}{p_t} - e^N .$$

Entrepreneurs

Like the foreign lenders, entrepreneurs are risk-neutral and derive utility from the consumption of tradable goods. Their preferences are given by the utility $U = c_1^T + \mathrm{E}_1[c_2^T]$.

Entrepreneurs have access to investment projects to produce non-tradable goods with a time-to-build technology. A project of size k requires investing a capital of k units of non-tradable goods and produces ak units of non-tradable goods in the next period. Capital completely depreciates during the production process. Once capital is installed, investment is not reversible. Depending on the game played by the entrepreneurs, investment projects can be divisible or indivisible.

Each entrepreneur enters period $t = 1$ with an already started investment project of size k_0 and a total debt $b_1^T + p_1 b_1^N$ to pay back during the period—where b_1^T (b_1^N) is a debt denominated in tradable (non-tradable) goods—and is endowed with a new project of size k_1. Denote $w_1 = p_1(ak_0 - b_1^N) - b_1^T$ her internal funds in the beginning of period 1.

[5] The riskless rate of return of bonds denominated in N goods is then equal to p_1/p_2 (uncovered interest parity).

In addition, in the FI game, each investment project can be hit by an idiosyncratic shock that requires some injection of international liquidity. This liquidity shock is modeled in the following way. Between periods $t = 1$ and $t = 2$, a random variable ξ is independently drawn for each entrepreneur in a set $\Xi \subset [0, +\infty)$ from a common knowledge cumulative distribution function F. The value of the shock ξ is only known at the end of period $t = 1$, after investment has taken place. The entrepreneur has then to provide a quantity ξk_1 of *tradable* goods to save the project, or else the initial investment is lost and the project does not yield any production. The required liquidity ξk_1 does not depreciate so that ξ represents a *pure* liquidity shock.

Financial contracts

During period $t = 1$, domestic entrepreneurs can borrow funds on the financial market on two occasions: before and after the liquidity shock. Denote d the amount borrowed before the shock is known to finance the initial investment and $d'(\xi)$ the amount borrowed after the shock is known to finance the required liquidity ξk_1. These loans are subject to a borrowing constraint. I assume that an entrepreneur can at most borrow $(\lambda - 1)$ times her internal funds, with $\lambda \geq 1$:

$$\forall \xi \in \Xi,\ d + d'(\xi) \leq (\lambda - 1) w_1 \,. \tag{1}$$

I shall refer to λ as the financial multiplier.

Technical assumptions

I conclude the description of the model by two technical assumptions on the exogenous parameters.

Assumption 1. $ak_0 > b_1^N + k_1$

This assumption ensures that domestic entrepreneurs are more liquid when the real exchange rate appreciates.

Assumption 2. $(a-1)e^N + a^2 k_0 > 0$

This assumption ensures that domestic investment is strictly positive when the financial system is perfect.

2.2 The Fear of Illiquidity game

In the FI game, investment projects are indivisible. Each entrepreneur of sector N decides whether she starts an investment project or puts her internal funds w_1 on the international financial market (or equivalently consumes them). Denote $x \in [0,1]$ the fraction of entrepreneurs who decide to start a project. Some of the projects started at $t = 1$ will have to be abandoned after the liquidity shock. Denote π the fraction of projects started at $t = 1$ that are not abandoned.

From the law of large numbers, the fraction π is also equal to the *ex ante* probability that a given entrepreneur is able to meet the liquidity shock. Because of the borrowing constraint (1), the maximum liquidity an entrepreneur can raise after the initial investment is equal to $\lambda w_1 - p_1 k_1$. The project is saved if this amount is greater than the needed liquidity ξk_1. This is the case when $\xi \leq \xi_l$, where the *liquidity threshold* ξ_l is defined by

$$\xi_l k_1 = \lambda w_1 - p_1 k_1 \,. \tag{2}$$

Therefore, the fraction π of saved projects is simply given by

$$\pi = F(\xi_l) \,. \tag{3}$$

Consider now the decision to start a project. Because of the borrowing constraint, an entrepreneur cannot invest more than λw_1. If $\lambda w_1 < p_1 k_1$, *i.e.* if $\xi_l < 0$, no entrepreneur can invest and we have $x = 0$. This initial borrowing constraint can therefore be written

$$x \xi_l \geq 0 \,. \tag{4}$$

Let us now write the market clearing conditions for non-tradable goods.

At $t = 1$, the supply of N goods is ak_0 and the demand is $D_1 + xk_1$. The market clears when

$$p_1 = \frac{d}{ak_0 - xk_1 + e^N} . \tag{5}$$

At $t = 2$, the supply of N goods is $x\pi ak_1$ and the demand is D_2. The market clears when

$$p_2 = \frac{d}{x\pi ak_1 + e^N} . \tag{6}$$

The equilibrium of the model can now be formally defined.

Definition 1. *An equilibrium of the FI game is defined by a price vector (p_1, p_2), a fraction of started projects x and a fraction of continued projects π that satisfy the initial borrowing constraint (4), the probability of continuation (3), the market clearing conditions (5) and (6), and such that, given the price vector, no entrepreneur can change its investment strategy to get a strictly higher utility.*

Note that nothing has been said about the bond market so far. Equilibrium in the bond market requires that the expected rate of return on bonds be equal to 1. As the liquidity shock is the only source of uncertainty, bonds issued after the shock to finance the required liquidity ξk_1 have a certain rate of return exactly equal to one. On the contrary, bonds issued before the shock are risky since they might not be fully repaid if the project is abandonned. Therefore, the rate of return R promised in case of success can be greater than one. Because of risk neutrality, entrepreneurs are indifferent between borrowing liquidity (*i.e.* external funds in excess of what is strictly needed to finance the initial investment) before or after the liquidity shock, so that d, $d'(\xi)$ and R are indeterminate. If we assume, to make things simple, that all the needed liquidity is borrowed after the shock is known, then $d = \max\big(p_1k_1 - w_1, 0\big)$ and $R = 1/\pi$.[6] In general, the rate of return R decreases with the probability of success π.

[6] In that case, the entrepreneur defaults on her loan and does not pay anything back when the project is abandonned. If, on the contrary, $d \geq p_1k_1 - w_1$ the creditor gets $d + w_1 - p_1k_1$ when the project is abandonned, which leads to a lower R given by $d = \pi Rd + (1 - \pi)(d + w_1 - p_1k_1)$.

2.3 The Constrained Investment game

In the CI game, projects are divisible and there is no liquidity shock. Therefore,

$$\pi = 1\,. \tag{3'}$$

Each entrepreneur chooses which fraction x of the project she starts at $t = 1$. The borrowing constraint is now given by:

$$p_1 x k_1 \leq \max(0, \lambda w_1)\,. \tag{4'}$$

Note that when $w_1 < 0$, the entrepreneur defaults on her past debt and cannot issue any new debt.

The analysis of the CI game is restricted to symmetric equilibria where all entrepreneurs choose the same strategy x. The formal definition is the following.

Definition 2. *An equilibrium of the CI game is defined by a price vector* (p_1, p_2) *and an investment strategy* x *that satisfy the initial borrowing constraint* (4') *and the market clearing conditions (5) and (6) for* $\pi = 1$, *and such that, given the price vector, no entrepreneur can change its investment strategy to get a strictly higher utility.*

2.4 Discussion of the assumptions

Liquidity shock in tradable goods

The assumption that the liquidity shock consists in providing *tradable* goods is essential. With this assumption, movements of the real exchange rate change the relationship between the cost of required liquidity and the entrepreneurs' internal funds, thereby creating a balance-sheet effect. More specifically, entrepreneurs, who produce non-tradable goods, are more likely to meet the liquidity shock when the real exchange rate is appreciated.

Irreversibility of investment and uncertainty

A key assumption of the model concerns the irreversibility of investment. If investment was reversible, it would be as if the decision to invest was taken after the liquidity shock ξ is known. The irreversibility of investment leads to the uncertainty of its return.

(In)divisible investment projects

In the FI game, investment projects have a fixed exogenous size. This assumption makes possible situations in which projects have a negative value so that entrepreneurs choose not to invest. As we shall see later, the possibility of such situations plays an important role in the kind of multiple equilibria the model can produce and in the conditions under which a crisis equilibrium exists.

Borrowing constraint

I have assumed that funds borrowed by entrepreneurs are limited by a borrowing constraint. This financial market imperfection limits the capacity of an entrepreneur to start an investment project. In the FI game, it can also prevent an entrepreneur from borrowing the liquidity needed to continue her project and thereby creates a situation of illiquidity.

The maximum size of a loan depends on internal funds. In the FI game, this assumption is inessential.[7] However, this specification is crucial in the CI game and was chosen to reproduce the structure of several recent models of financial crises in open economies (Krugman 1999, Schneider & Tornell 2004, Aghion et al. 2004*a*, Aghion, Bacchetta & Banerjee 2004*b*).

In this literature, the financial multiplier λ is interpreted as an indicator of development of the domestic financial system. The case $\lambda = \infty$ corresponds to a perfect financial system with no constraint. The polar case

[7] It could have been equally assumed that the loan is limited by some exogenous collateral E, as in Caballero & Krishnamurthy (2001). The constraint (1) would then simply be $d + d' \leq E$. The important point is that the constraint should not weaken when the real exchange rate depreciates.

$\lambda \to 1$ corresponds to full financial repression where firms cannot issue any debt.

Endowments of the domestic consumers

I have assumed that domestic consumers are endowed with some non-tradable goods. The existence of this endowment ensures that the market for N goods clears at a finite price p_2 in period $t = 2$, even if no entrepreneur invests at $t = 1$.

3 Equilibria of the model

3.1 The investment strategy of entrepreneurs

Each entrepreneur takes p_1 and p_2 as given when she chooses her strategy. From the market clearing conditions (5) and (6), this amounts to taking x and π as given. In the CI (FI) game, x represents the strategy played by *other* entrepreneurs (the fraction of *other* entrepreneurs starting a project). In the following, I use the notations $p_1(x)$, $p_2(x, \pi)$, $w_1(x)$ and $\xi_l(x)$ to make this dependence clear.

Let us now determine the constrained optimal strategy of a given entrepreneur. The expected net value of the project is equal to $V(x, \pi)k_1$ where

$$V(x, \pi) = -p_1(x) + \pi p_2(x, \pi)a\,. \tag{7}$$

As the creditors get an expected return equal to one, the value of the project is completely captured by the entrepreneur.

In the FI game, the entrepreneur's strategy consists in deciding whether she starts the project or not. If $\xi_l(x) < 0$, she is not able to finance the initial capital and therefore cannot start the project. If $\xi_l(x) \geq 0$, three cases have to be considered.

- When $V(x, \pi) > 0$, the entrepreneur starts the project and gets an expected utility $w_1(x) + V(x, \pi)k_1$.

- When $V(x,\pi) < 0$, the entrepreneur does not start the project at all. Instead, she puts her internal funds $w_1(x)$ on the financial market with a rate of return equal to one, or equivalently consumes them, and gets a utility equal to $w_1(x)$.

- When $V(x,\pi) = 0$, the entrepreneur is indifferent between investing in the project or not.

In the CI game, the entrepreneur's strategy is the fraction of the project she decides to start.

- When $V(x,\pi) > 0$, the entrepreneur starts the highest fraction compatible with the initial borrowing constraint (4'), *i.e.* $\min\Big(1, \max\big(0, \lambda\frac{w_1(x)}{p_1(x)k_1}\big)\Big)$.

- When $V(x,\pi) < 0$, the entrepreneur does not start the project at all.

- When $V(x,\pi) = 0$, the entrepreneur is indifferent as to the fraction of the project she decides to start. The set of optimal strategies is $\Big[0, \min\Big(1, \max\big(0, \lambda\frac{w_1(x)}{p_1(x)k_1}\big)\Big)\Big]$.

3.2 The case of a perfect financial system

Let us briefly turn to the case of a perfect financial system with $\lambda = \infty$. Then the borrowing constraints (4) or (4') never bind. Moreover, in the FI game, the entrepreneur can always borrow enough liquidity to meet the liquidity shock so that $\pi = 1$. Therefore, the CI game and the FI game are formally equivalent when $\lambda = \infty$.

Claim 1 (Equilibrium with a perfect financial system)**.** *When $\lambda = \infty$, both the CI and the FI game have a unique equilibrium $x^* \in (0,1]$.*

Proof. From the discussion in section 3.1, an equilibrium of the model without borrowing constraint falls in one of these three cases: (a) $x = 0$ with $V(0,1) < 0$ (no entrepreneur invests), (b) $x = 1$ with $V(1,1) > 0$ (entrepreneurs start the whole project in the CI game and all entrepreneurs invest in the FI game), and (c) $0 \leq x \leq 1$ with $V(x,1) = 0$ (all entrepreneurs start a fraction x of the project in the CI game and only a fraction x of entrepreneurs invest in the FI game, entrepreneurs being indifferent between investing or not since the project has a zero net expected value).

The first case is ruled out because $V(0,1) > 0$ under assumption 2. From (5), $p_1(x)$ is a strictly increasing function and from (6), $p_2(x,1)$ strictly decreases with x. Therefore, $V(x,1)$ is a strictly decreasing function of x. Besides, $p_1(x) \longrightarrow +\infty$ and therefore $V(x,1) \longrightarrow -\infty$ when x increases toward $(ak_0 + e^N)/k_1$. From the continuity of $V(x,1)$, there is a unique $\tilde{x}$ in $\big(0, (ak_0 + e^N)/k_1\big)$ such that $V(\tilde{x},1) = 0$. If $\tilde{x} \leq 1$, we are in case (c) and $\tilde{x}$ is the unique equilibrium of the model. If $\tilde{x} > 1$, then $V(1,1) > 0$ and we are in case (b).

Therefore, the model has a unique equilibrium $x^* = \min(\tilde{x}, 1)$. □

The uniqueness of the equilibrium when $\lambda = \infty$ comes from the fact that the value of an individual project $V(x,1)k_1$ decreases when the aggregate investment x increases. In other words, there is *strategic substitutability* between the investment decisions of entrepreneurs.

To make things interesting, I assume that the entrepreneurs are able to repay their past debt in the unique equilibrium x^*, *i.e.* that $w_1(x^*) > 0$.

Assumption 3. *The past debts b_1^T and b_1^N are such that*

$$b_1^T < \frac{d(ak_0 - b_1^N)}{ak_0 - x^* k_1 + e^N}\,. \tag{8}$$

3.3 Multiple equilibria in the CI game

This section studies the equilibria of the CI game. Results presented here are similar to those of existing models based on the inability to invest during balance-of-payments crises.

An equilibrium is a value of x which is a fixed point of the constrained optimal strategy described in section 3.1. Graphically, define the investment schedule in $[0,1]^2$ as the schedule that associates to each value of x in $[0,1]$ the corresponding set of constrained optimal strategies. To be an equilibrium, x has to belong to the set of constrained optimal strategies associated to itself. In other words, equilibria are given by intersections of the investment schedule with the 45 degree line.

Equilibria can be of three types: (a) unconstrained equilibria H given by $x^H = x^*$ (where x^* was defined in section 3.2) and such that the initial borrowing constraint (4') does not bind; (b) constrained equilibria C with no default, such that the initial borrowing constraint (4') binds but $w_1(x^C) > 0$; (c) constrained equilibria D with defaults, with $x^D = 0$ and such that $w_1(0) < 0$.

To restrict the set of equilibria, I impose a stability condition.[8]

Definition 3. *An equilibrium is said to be stable if an infinitely small departure of x above (below) its equilibrium value would induce agents to change their strategy in a way that would make x decrease (increase).*

With this definition, an equilibrium of the CI game is stable when the slope of investment schedule at the equilibrium is strictly lower than 1.

Figure 1 shows an example where there are several simultaneous equilibria: an unstable equilibrium C along with two stable equilibria D and H. From the market clearing condition (5), the real exchange rate is more

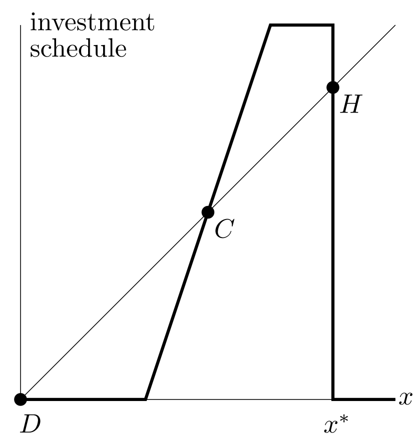

Figure 1: Multiple equilibria in the CI game

appreciated in equilibrium H than in equilibrium D. Besides, aggregate

[8]This definition of stability corresponds to the stability of any intuitive out-of-equilibrium adjustment process.

investment is high in equilibrium H (it is equal to $x^* p_1(x^*) k_1$ in value) whereas there is no investment and entrepreneurs default on their past debt in equilibrium D. Thus, equilibrium D can be identified to a situation of balance-of-payments crisis and equilibrium H to tranquil times.

The simultaneous existence of these two stable equilibria accounts well for the possibility of sudden balance-of-payments crises with large real depreciations and significant decreases in aggregate investment. Such a crisis can be triggered by a purely expectational shock (a *sunspot*) or by any unexpected exogenous shock that can lead to the disappearance of the good equilibrium, *e.g.* a negative shock in the demand for non-tradable goods (decrease in d) or a tightening of the borrowing constraint (decrease in λ).

The following result formally derives conditions for the existence of multiple stable equilibria.

Claim 2 (Multiple equilibria in the CI game)**.** *The CI game has a stable unconstrained equilibrium H characterized by $x^H = x^*$ if and only if*

$$\lambda^{-1} d < b_1^T + \frac{1}{x^* k_1} \left[d(ak_0 - b_1^N) - b_1^T (ak_0 + e^N) \right] . \tag{9}$$

The CI game has a stable constrained equilibrium D with defaults characterized by $x^D = 0$ if and only if

$$b_1^T > d \, \frac{ak_0 - b_1^N}{ak_0 + e^N} \, . \tag{10}$$

The CI game has multiple stable equilibria if and only if both conditions (9) *and* (10) *are satisfied. Then, there are two stable equilibria of type H and D.*

Proof. Condition (9) is equivalent to $x^* p_1(x^*) k_1 < \lambda w_1(x^*)$. Under assumption 3, it just states that the initial borrowing constraint (4') does not bind for x^*, which is the only unconstrained equilibrium from Claim 1. Condition (10) is equivalent to $w_1(0) < 0$, a sufficient and necessary condition for the existence of a constrained equilibrium with defaults and no investment. These two equilibria are clearly stable in the sense of Definition 3.

It remains to be proven that multiple stable equilibria are necessarily of type H and D. Suppose there is an equilibrium of type C, *i.e.* an equilibrium x^C where the initial borrowing constraint binds but the entrepreneurs do not default, and let us show that the stability of this equilibrium entails its uniqueness. The strategy x^C is a zero of $g(x) = \lambda \frac{w_1(x)}{p_1(x)k_1} - x$ and $x^C \in (0, x^*)$. Because the function $g(x)$ is linear in x—see equation (5)— the stability condition means that $g(x)$ is strictly decreasing. Therefore $g(0) > 0$ so that $x^D = 0$ is not an equilibrium and $g(x^*) < 0$ so that x^* is not an equilibrium. □

The following corollary is a direct consequence of Claim 2.

Corollary 3. *A necessary condition for the existence of multiple stable equilibria is that*

$$\lambda b_1^T > d\,. \tag{11}$$

Proof. Condition (11) is implied by conditions (9) and (10) when they hold simultaneously. □

This corollary states that a situation of multiple equilibria requires a large enough debt in tradable goods b_1^T, i.e. a sufficient degree of currency mismatch, and a strong enough financial multiplier λ, *i.e.* a sufficiently developed financial system. This result is in line with findings from Krugman (1999), Schneider & Tornell (2004) and Aghion et al. (2004*a*). The currency mismatch is necessary to get defaults when no entrepreneur invests, a necessary characteristic of the crisis equilibrium D. The high value of the financial multiplier creates a strong financial accelerator effect which makes the intermediary equilibrium C unstable[9] and allows the existence of two other stable equilibria. Intuitively, when investment x decreases out-of-equilibrium, it provokes a decrease in the relative price $p_1(x)$, which diminishes the internal funds $w_1(x)$ and strengthens the borrowing constraint. For a strong enough financial multiplier, the induced decrease in x is large enough to accelerate the downward movement of x and $p_1(x)$ until entrepreneurs default and $x = 0$.

[9] Condition (11) exactly states that equilibrium C is unstable.

These formal properties of the CI game have interesting consequences in the limit cases of full financial repression and almost perfect financial system.

Corollary 4. *When* $\lambda = 1$*, the CI game has a unique equilibrium.*

When $\lambda \to \infty$ *with* $\lambda < \infty$*, inequality* (10) *is a necessary and sufficient condition for the existence of multiple equilibria.*

Proof. To prove the first part of this corollary, let us show that H and D cannot both be equilibria when $\lambda = 1$. To do this, just note that $w_1(x) - xp_1(x)k_1$ is decreasing with x under assumption 1 so that $w_1(0) > w_1(x^*) - x^*p_1(x^*)k_1$. Therefore, if D is an equilibrium, the initial borrowing constraint binds for x^*, so that H is not an equilibrium. On the contrary, if H is an equilibrium, we have $w_1(0) > 0$ and D is not an equilibrium.

The second part of the corollary comes from the fact that condition (9) is equivalent to condition (8) when $\lambda \to \infty$, which is true under assumption 3.

□

Let us summarize the findings of this section:

- In the CI game where investment projects are divisible, there can be two stable equilibria associated to different levels of aggregate investment and real exchange rate. Entrepreneurs are unable to invest in the crisis equilibrium.

- The crisis equilibrium has to be an equilibrium where entrepreneurs default on their past debt, which requires a strong enough currency mismatch (a large enough debt in tradable goods).

- The multiplicity of equilibria also requires a large enough financial multiplier. Thus, there can be no crisis in an economy subject to full financial repression, while a crisis is possible when the financial system is almost perfect.

3.4 Strategic complementarity and multiple equilibria in the FI game

Consider now the FI game where investment projects are indivisible and there is a liquidity shock. This game also has equilibria with constrained investment. More precisely, if $\xi_l(0) < 0$, $x = 0$ is an equilibrium where no entrepreneur invests because the constraint (4) is binding. A sufficient condition for this is that $w_1(0) < 0$ (entrepreneurs default on their past debt). Note that the existence of a constrained equilibrium requires a strictly positive debt denominated in tradable goods b_1^T.[10]

The remaining of this section focuses on equilibria where investment is not constrained, *i.e.* equilibria characterized by a proportion x such that $\xi_l(x) \geq 0$.

In the FI game, the probability of success of an investment project π is given by (3). Denote $\pi(x) = F\big(\xi_l(x)\big)$. Unconstrained equilibria fall into one these three cases: (a) $x = 0$ with $V\big(0, \pi(0)\big) < 0$, (b) $x = 1$ with $V\big(1, \pi(1)\big) > 0$, and (c) $0 \leq x \leq 1$ and $V\big(x, \pi(x)\big) = 0$.

Using Definition 3, equilibria of type $x = 0$ (constrained or not) and $x = 1$ are stable, and equilibria of type $V\big(x, \pi(x)\big) = 0$ are stable when the function $x \mapsto V\big(x, \pi(x)\big)$ is strictly decreasing around the equilibrium value of x.

As in the case of a perfect financial system (see section 3.2), the mapping $x \mapsto V(x, \pi)k_1$ is strictly decreasing, suggesting that there might be strategic substitutability. However, the mapping $\pi \mapsto V(x, \pi)$ is strictly increasing: *cet. par.* the value of a project increases with its probability of success.[11] Besides, the liquidity threshold ξ_l strictly increases with the relative price p_1 under assumption 1: because the revenue of entrepreneurs consists of non-tradable goods, they are more liquid when the real exchange rate appreciates (provided that the cost of investment does not increase too

[10] This is obvious for the case $w_1(0) \leq 0$. When $w_1(0) > 0$, $\xi_l(0)k_1 \geq w_1(0) - p_1(0)k_1 = p_1(0)(ak_0 - b_1^N - k_1) - b_1^T$. From assumption 1 this can only be strictly negative if b_1^T is strictly positive.

[11] This comes from the fact that $\dfrac{\partial V}{\partial \pi} = a\dfrac{\partial}{\partial \pi}(\pi p_2) = ap_2\dfrac{e^N}{x\pi a k_1 + e^N} > 0$.

much, which is ensured by assumption 1). Therefore, from equation (3), $\pi(x)$ is an increasing function of x so that $V(x, \pi(x))$ might also be increasing with x. This is a potential source of *strategic complementarity*: as more entrepreneurs choose to invest, investment demand for non-tradable goods increases, the real exchange rate appreciates, entrepreneurs become more liquid and the probability of success of their project increases, which might increase the value of the project. If this second effect dominates and the resulting strategic complementarity is strong enough, the FI game can produce situations of multiple equilibria where investment is unconstrained.[12]

Figure 2 shows with a few examples that this can be the case for different cumulative distribution functions $F(\xi)$. Intuitively, multiple unconstrained equilibria can arise when the liquidity threshold $\xi_l(x)$ takes its values in an interval where the cumulative distribution function F increases quickly and strongly enough. In the examples displayed in figure 2, there are two stable unconstrained equilibria, labeled H and L. Equilibria H can be interpreted as tranquil time equilibria while equilibria L correspond to crisis times. In general there can be an arbitrary (even) number of stable equilibria.[13]

As in section 3.3, equilibria with a higher value of x are characterized by a more appreciated real exchange rate and a higher aggregate investment, and vice versa. However, in crisis-type equilibria—equilibria L in the examples of figure 2— the low level of investment is not the result of a binding initial constraint but of the unwillingness of entrepreneurs to invest. This unwillingness is motivated by the increased uncertainty on their ability to meet future liquidity shocks if they start the project. Because of this increased uncertainty, the *ex ante* value of a project is negative and entrepreneurs prefer to put their internal funds on the international financial market (or equivalently to consume them) instead of investing. Thus, expectations of entrepreneurs about their likelihood to run a project to its completion prove to be a powerful amplification mechanism of real exchange rate movements.

Contrary to the "inability to invest" mechanism described in section 3.3,

[12]See Cooper & John (1988) for a generic theoretical framework on strategic complementarities and multiple equilibria.

[13]This can be the case if the distribution function of the shocks has several modes.

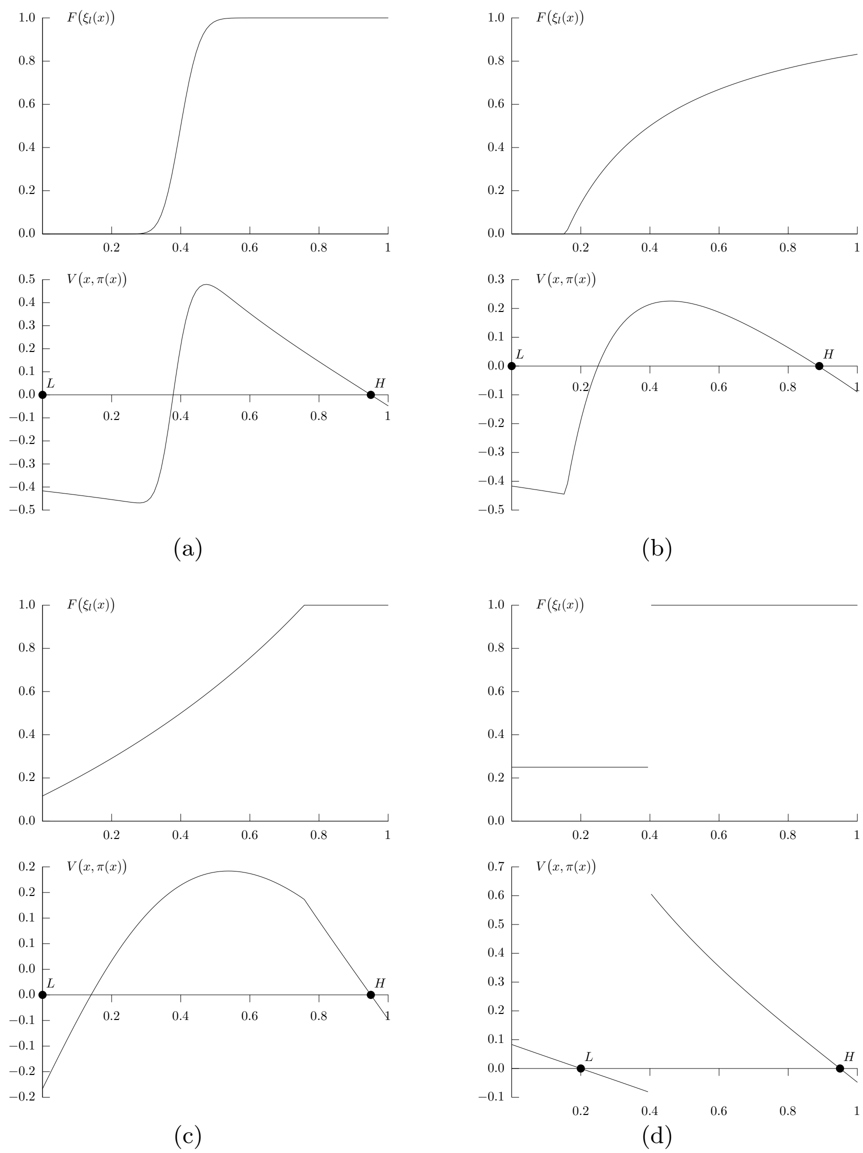

Figure 2: Examples of unconstrained multiple equilibria in the FI game for different distributions of the liquidity shocks. (a) Log-normal distribution with $\log(\xi) \sim N(\log(0.1), 0.04)$. (b) Pareto distribution with $F(\xi) = 1 - 0.05/\xi$ for $\xi \geq 0.05$. (c) Uniform distribution of support [0,0.2]. (d) Binomial distribution: $\xi = 0$ or 0.1 with probabilities 0.25 and 0.75. The other parameters are $d = k_0 = k_1 = 1$, $a = 1.6$, $e^N = 0.8$, $\lambda = 1.2$, $b_1^T = 0.3$, and $b_1^N = 0$. For these parameters $\xi_l(x) > 0$ for all $x \in [0, 1]$ so that the initial borrowing constraint never binds. The two stable equilibria are labeled H and L.

it is clear that this "unwillingness to invest" mechanism requires a small enough financial multiplier λ and is only relevant in countries with a low enough level of financial development. When $\lambda \to \infty$ with $\lambda < \infty$, the probability of continuation π gets infinitely close to 1 and there only remains one unconstrained equilibrium (which is close to x^*). Then, as in section 3.3, the only possible crisis equilibrium is given by $x = 0$: it exists if and only if $w_1(0) < 0$—*i.e.* if there is a strong enough currency mismatch—and it is characterized by the inability of entrepreneurs to invest.

The next section studies in more details the case of a binomial distribution.

4 The case of a binomial distribution of liquidity shocks

Consider the following distribution of the liquidity shock ξ for the FI game:

$$\xi = \begin{cases} 0 & \text{with probability } q \\ \bar{\xi} & \text{with probability } 1-q \end{cases}$$

with $q \in (0,1)$.

Then the probability of success of a given entrepreneur is either $\pi(x) = 1$ if $\xi_l(x) \geq \bar{\xi}$ or $\pi(x) = q$ if $0 \leq \xi_l(x) < \bar{\xi}$. Besides, if $\xi_l(x) < 0$, the initial borrowing constraint binds and the entrepreneur cannot start the project.

Define $V^H(x) = V(x,1)$ and $V^L(x) = V(x,q)$. A candidate for an unconstrained stable tranquil time equilibrium, denoted H, is given by $x^H = x^*$, where x^* is the unique equilibrium that exists in the absence of a borrowing constraint (see section 3.2). Denote $\tilde{x}^L$ the unique root of V^L in the interval $\big(-e^N/(qak_1), (ak_0 + e^N)/k_1\big)$.[14] Let $x^L = \min\big(1, \max(0, \tilde{x}^L)\big)$. The strategy x^L is a candidate for an unconstrained stable crisis equilibrium L.

If $w_1(x^L) < 0$, all entrepreneurs default when $x = x^L$ and x^L can never be an equilibrium, whatever the values of λ and $\bar{\xi}$. Furthermore, it is possible

[14]The function $V^L(\cdot)$ is continuous and strictly decreasing. It goes to $-\infty$ when x goes to $(ak_0 + e^N)/k_1$ and to $+\infty$ when x goes to $-e^N/(qak_1)$.

that $x^L = x^H = x^* = 1$, in which case there is a unique equilibrium $x = 1$ that can be either of type H or of type L depending on the sign of $\xi_l(1) - \bar{\xi}$. Assume that none of this is the case:

Assumption 4. *The probability q is such that*

$$p_1^{-1}\left(\frac{b_1^T}{ak_0 - b_1^N}\right) < x^L < x^* .$$ [15]

The left hand-side inequality is equivalent to $w_1(x^L) > 0$. The right hand-side inequality is possible because x^L can be arbitrarily small for sufficiently small values of q. Both inequalities can be simultaneously satisfied because $w_1(\cdot)$ is continuous and $w_1(x^*) > 0$ from assumption 3.

Then, a situation where both x^H and x^L are unconstrained stable equilibria of the FI game arise for some values of λ and $\bar{\xi}$.

Claim 5. *If $w_1(x^L) \leq p_1(x^L)k_1$, the strategies x^H and x^L are both unconstrained and stable equilibria of the FI game in the following non-empty region of the quadrant $\left\{\lambda \geq 1, \bar{\xi} > 0\right\}$:*

$$\bar{\xi} > 0$$
$$\max\left(\frac{p_1(x^L)k_1}{w_1(x^L)}, \frac{\bar{\xi} + p_1(x^H)k_1}{w_1(x^H)}\right) \leq \lambda < \frac{\bar{\xi} + p_1(x^L)k_1}{w_1(x^L)} .$$

If $w_1(x^L) > p_1(x^L)k_1$, the strategies x^H and x^L are both unconstrained and stable equilibria of the FI game in the following non-empty region of the quadrant $\left\{\lambda \geq 1, \bar{\xi} > 0\right\}$:

$$\bar{\xi} > w_1(x^L) - p_1(x^L)k_1$$
$$\max\left(1, \frac{\bar{\xi} + p_1(x^H)k_1}{w_1(x^H)}\right) \leq \lambda < \frac{\bar{\xi} + p_1(x^L)k_1}{w_1(x^L)} .$$

Proof. The strategy x^L is an unconstrained equilibrium if and only if $0 \leq \xi_l(x^L) < \bar{\xi}$ and the strategy x^H is an unconstrained equilibrium if and only

[15] The notation $p_1^{-1}(\cdot)$ represents the inverse function of $p_1(\cdot)$, *i.e.* the function $\varphi(p_1)$ such that $\varphi(p_1(x)) = x$.

if $\xi_l(x^H) \geq \bar{\xi}$.

The inequality $\xi_l(x^H) \geq \bar{\xi}$ is equivalent to $\frac{\bar{\xi}+p_1(x^H)k_1}{w_1(x^H)} \leq \lambda$. The inequality $\xi_l(x^L) < \bar{\xi}$ is equivalent to $\lambda < \frac{\bar{\xi}+p_1(x^L)k_1}{w_1(x^L)}$. Given that λ must be greater than or equal to 1, this requires that $\bar{\xi} > w_1(x^L) - p_1(x^L)k_1$ when $w_1(x^L) > p_1(x^L)k_1$.

Besides, the inequality $\xi_l(x^L) \geq 0$ is equivalent to $\frac{p_1(x^L)k_1}{w_1(x^L)} \leq \lambda$, which is always true when $w_1(x^L) > p_1(x^L)k_1$.

Finally, the interval $\left[\frac{\bar{\xi}+p_1(x^H)k_1}{w_1(x^H)}, \frac{\bar{\xi}+p_1(x^L)k_1}{w_1(x^L)}\right)$ is non-empty because $w_1(x)/p_1(x)$ increases with x and $w_1(x)$ strictly increases with x. □

See figure 3 for a graphical illustration of this claim.

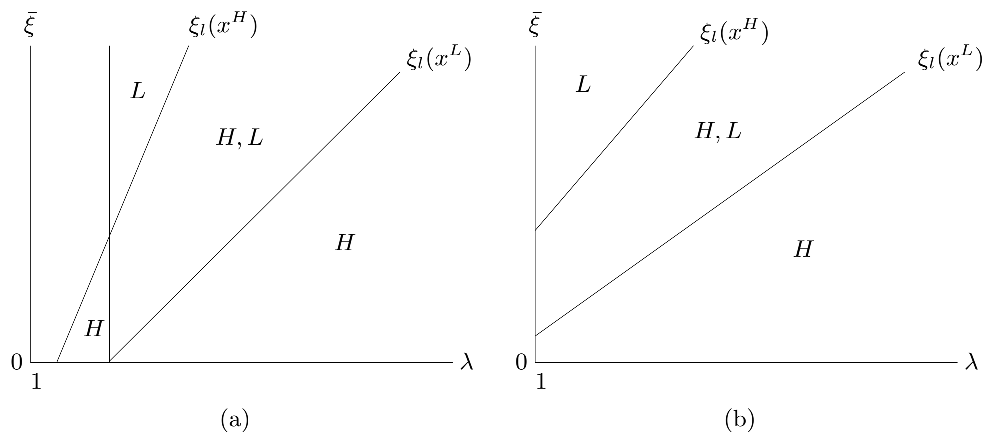

Figure 3: Unconstrained equilibria in the FI game for a binomial distribution ($\xi = 0$ or $\bar{\xi}$ with probabilities q and $1-q$). (a) Case $w_1(x^L) \leq p_1(x^L)k_1$. (b) Case $w_1(x^L) > p_1(x^L)k_1$.

This claim shows that, provided the liquidity shock $\bar{\xi}$ is large enough, there are multiple unconstrained equilibria for intermediate levels of the financial multiplier λ. If λ is too large, there is no crisis equilibrium and only the tranquil time equilibrium H exists. If λ is too small, either the tranquil time strategy x^H is not an equilibrium (because the associated probability of success is too small) or the crisis time strategy x^L is not an equilibrium because entrepreneurs cannot start the project.

However, note that when $w_1(x^L) \geq p_1(x^L)k_1$ (cf. figure 3b) and $w_1(x^L)-$

$p_1(x^L)k_1 < \bar{\xi} \leq w_1(x^H) - p_1(x^H)k_1$, $\lambda = 1$ falls into the region where the two equilibria H and L simultaneously exist. Thus, this crisis mechanism which relies on the unwillingness of entrepreneurs to invest is also relevant for economies with a very low level of financial development. Also note that, contrary to the "inability to invest" mechanism, the debt denominated in tradable goods b_1^T need not be large here. In fact, multiple unconstrained equilibria are even possible with $b_1^T = 0$.

As noted earlier, the FI game also has equilibria where investment is constrained and $x = 0$. Because investment is indivisible in the FI game, these constrained equilibria have different properties than in the CI game. In particular, a stable equilibrium C with constrained investment but no default can now coexist with an unconstrained equilibrium even when the financial multiplier λ gets close to 1.[16]

Here is a summary of the properties of the FI game.

- In the FI game where investment projects are indivisible and there is a liquidity shock, there can be a large multiplicity of stable equilibria where non-investing entrepreneurs are not constrained.

- A necessary condition for the existence of multiple unconstrained equilibria is that the financial multiplier λ be sufficiently small. Multiple unconstrained equilibria are compatible with both low levels of financial development and a low degree of currency mismatch.

- When some fraction of the past debt is denominated in tradable goods, an equilibrium where no entrepreneurs invest because of a binding borrowing constraint can exist along with unconstrained equilibria.

[16] Consider for example the limit case of full financial repression where $\lambda = 1$. When $0 \leq w_1(0) < p_1(0)k_1$ and $w_1(x^H) - p_1(x^H)k_1 \geq \bar{\xi}k_1$, which is possible because $w_1(x) - p_1(x)k_1$ is increasing with x, there are at least two stable equilibria: the unconstrained equilibrium H and a constrained equilibrium C with $x^C = 0$ and no default.

5 Conclusion

The aim of this paper was to explore another channel through which a real depreciation leads to a decrease in investment during a balance-of-payments crisis. In the existing literature, the drop in investment is explained by a binding borrowing constraint that prevents domestic entrepreneurs to invest. In this paper, I have shown that it can also be the result of their *unwillingness* to invest. When investment projects are hit by international liquidity shocks and domestic entrepreneurs have limited collateral, a real depreciation diminishes their liquidity and decreases the probability of success of their investment projects. With indivisible projects, starting a project can then be less profitable than putting their internal funds on the international financial market. The paper has studied this mechanism, along with the traditional *inability*-to-invest channel, in a small open economy where crises are modeled by the existence of multiple equilibria.

The crisis mechanism based on the unwillingness to invest differs from the inability-to-invest channel by several aspects. To begin with, the observed decrease in lending comes from a drop in demand, not in supply. This is consistent with the recent empirical evidence from Hale & Arteta (2006) discussed in the introduction. It is also consistent with the strong recovery of investment observed during emerging market crises, which seems to be neither financed by domestic credit nor by foreign capital flows, suggesting that the collapse of investment during crises cannot be entirely explained by a falling supply of loanable funds (Calvo et al. 2006).

A second difference concerns the intensity of the borrowing constraint. As it was shown in the paper, the mechanism studied by the existing literature requires a sufficiently weak borrowing constraint and is best suited for economies with a high enough level of financial development. On the contrary, self-fulfilling crises based on the unwillingness to invest require a sufficiently strong borrowing constraint and are possible in economies subject to (possibly full) financial repression. Thus, this alternative mechanism should be relevant for emerging economies where firms have a difficult access to finance.

Last of all, while the existence of a low investment equilibrium with constrained domestic entrepreneurs requires the presence of a currency mismatch, it is not a necessary condition for the existence of a crisis equilibrium where entrepreneurs are unconstrained. In the latter case, the balance-sheet effect comes from the fact that the domestic production process might require at some point the use of foreign inputs.

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